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On the Effect of Absorption of Gravitation†

DIERCK-EKKEHARD LIEBSCHER

Deutsche Akademie der Wissenschaften, Sternwarte Babelsberg

Abstract

In the tetrad theory of gravitation Treder recently proposed field equations with potential coupling of matter. This coupling effects an absorption of the sources by the field itself. The changing value of solar potential along the elliptical path of the earth around the sun causes a change of the active gravitational mass of the earth with the annual period, so that, for instance, a pendulum will show an annual change in frequency. This effect may be tested, when the accuracy of measurements of the field of the earth has slightly increased.

1. *Introduction: Treder's Tetrad Theory of Gravitation*

Only the postulation of a tetrad field allows the straightforward introduction of spinors in Riemann spaces. This requires discussion about a tetrad formulation of gravitation theory. A tetrad field manifests itself as an anholonomic transformation of Minkowski space into Riemann space (Treder, 1966):

$$\eta_{AB}\, h^A{}_\mu(x)\, h^B{}_\nu(x) = g_{\mu\nu}(x) \tag{1.1}$$

η_{AB} is the Minkowski tensor of signature −2.
The metric spinors are linear in the tetrad components:

$$\sigma_\mu{}^{\Gamma\dot\Delta}(x) = h^A{}_\mu(x)\, \sigma_A{}^{\Gamma\dot\Delta} \tag{1.2}$$

$\sigma_A{}^{\Gamma\dot\Delta}$ are the Pauli spinors.

Given $g_{\mu\nu}$, equation (1.1) determines the tetrad only up to local Lorentz transformations, i.e. Lorentz transformations with variable coefficients. If $h^A{}_\mu = \underset{0}{h}{}^A{}_\mu$ is a solution of (1.1), then

$$h^A{}_\mu = \omega^A{}_B(x)\, \underset{0}{h}{}^B{}_\mu(x), \qquad \omega^A{}_B(x)\, \omega^C{}_D(x)\, \eta_{AC} = \eta_{BC} \tag{1.3}$$

is also a solution. If the tetrad is only fixed up to local Lorentz transformations, the metric contains all physical information of the tetrad, and the theory is a metric theory in redundant representation. On the other hand, equation (1.2) fixes the tetrad, given $\sigma_\mu{}^{\Gamma\dot\Delta}(x)$, entirely. But because of the

† *Greek minuscules*, running from 0 to 3, are indices of the Riemann space. *Latin minuscules* take the values 1, 2, 3. *Latin capitals*, running from 0 to 3, are indices of the tangent space. *Greek capitals* take the values 1 and 2 and are indices of the unitary spinor space.

unitary invariance of the spinor field equations, $\sigma_\mu{}^{\Gamma\dot{\Delta}}$ is given only up to these unitary transformations, which translate into the freedom of the tetrad with respect to global Lorentz transformation:

$$\underset{0}{h^A}_\mu(x) \quad \text{and} \quad \omega^A{}_B \underset{0}{h^B}_\mu(x), \quad \omega^A{}_B\, \omega^C{}_D\, \eta_{AC} = \eta_{BD} \tag{1.4}$$

are physically equivalent, if the $\omega^A{}_B$ are constant (Treder, 1966, 1967a).

To introduce spinor fields, we have to fix the tetrad up to these global Lorentz transformations. This done we have chosen a global parallelism in the Riemann space by the following possibility of definition:

$u^\mu(x_1)$ is equal to $u^\mu(x_2)$, exactly, if

$$u^\mu(x_1)\, h^A{}_\mu(x_1) = u^\mu(x_2)\, h^A{}_\mu(x_2) \tag{1.5}$$

This kind of definition is invariant only with respect to global Lorentz transformation of the tetrad (Einstein, 1928).

As far as only tensorial fields are considered in connection with gravity, only the knowledge of the metric $g_{\mu\nu}$ is required to describe the influence of gravitation. But one has to consider also spinor fields. Therefore the formulation of equations determining the tetrad field up to global Lorentz transformations has to be discussed. In analogy with the covariant Heisenberg equation (Heisenberg, 1967)

$$i\sigma^\mu{}_\Gamma{}^{\dot{\Delta}}(\psi_{\dot{\Delta},\mu} - \tfrac{1}{2}\sigma_{\nu\theta\dot{\Delta};\mu}\,\sigma^{\nu\theta\dot{H}}\,\psi_{\dot{H}}) + l^2\,\psi^H\,\sigma_{\mu H}{}^{\dot{\theta}}\,\psi_{\dot{\theta}}\,\sigma^\mu{}_\Gamma{}^{\dot{\Delta}}\,\psi_{\dot{\Delta}} = 0$$

$$i\sigma^\mu\, D_\mu\psi + l^2 j_\mu\,\sigma^\mu\psi = 0 \tag{1.6}$$

Weyl operator $\times$ field + current density $\times$ field = 0

Treder proposed the following equations for the tetrad (Treder, 1967a, b):

$$\alpha^{\rho\sigma}\, h^A{}_{\mu\,\|\,\rho\sigma} + \kappa(T_\mu{}^\nu - \tfrac{1}{2}\delta_\mu{}^\nu\, T_\rho{}^\rho)\, h^A{}_\nu = 0 \tag{1.7}$$

d'Alembert operator $\times$ field + matter density $\times$ field = 0

The matter tensor has to be corrected to the form $T^{*\nu}_\mu = T_\mu{}^\nu - \frac{1}{2}\delta_\mu{}^\nu T_\rho{}^\rho$ to secure the Newton limit for hydrodynamics. The form of the d'Alembert operator is essential for the theory. The metric $\alpha_{\mu\nu}$ ($\alpha_{\mu\nu}\alpha^{\nu\rho} = \delta_\mu{}^\rho$) represents a flat underground, the covariant differentiation denoted by $\|$ is constructed by the Christoffel symbols of this metric. The flat underground metric has to be introduced, if we want the differential term in the equation to be linear in the tetrad components. That is required in analogy to the first term in the Heisenberg equation, which is linear in ψ. The use of $g_{\mu\nu}$ in constructing the d'Alembert would mean nonlinearity because of (1.1), and that was intended to be avoided. There would be also difficulties with the general covariance of the solutions, imposing more restrictions on the matter tensor $T_\mu{}^\nu$ than the dynamical equation (Treder, 1967c). Equation (1.7) is chosen for the covariant components of the tetrad because of the fundamental role of the metric tensor and (1.1). Written for the contravariant components of the tetrad, equation (1.7) represents another theory (Treder, 1967a).

The Treder equation (1.7) is covariant with respect to general coordinate transformations and to global Lorentz transformations of the tetrad field. The weak equivalence principle represents a restriction for the field equations of the non-gravitational fields. They are responsible for the validity of the dynamical equation

$$T_{\mu\ ;\nu}^{\ \nu} = 0 \tag{1.8}$$

which is one formulation of the weak equivalence principle. The strong equivalence principle is not valid.

The flat underground metric $\alpha_{\mu\nu}$ influences only the tetrad field equations, all other equations consider only the tetrad and the metric $g_{\mu\nu}$ of the Riemann space. This results in the fact that in this theory gravitational waves propagate along the null cone of the flat underground metric, whereas the other fields propagate along the null cone of the Riemann metric $g_{\mu\nu}$. Therefore gravitational waves do not propagate with the velocity of light, in general they propagate faster: If ϕ denotes the Newton potential divided by c^2, it yields in the weak field

$$c_{\text{light}} \sim (1 - \phi)\, c_{\text{vac.}}, \qquad c_{\text{grav.}} = c_{\text{vac.}}$$

As the metric $g_{\mu\nu}$ can be measured by light signals and normal clocks, the flat underground metric can be explored, at least in principle, by the propagation of gravitational waves (Treder, 1967a).

The static spherical solution outside the source has the form

$$\alpha_{\mu\nu} = \eta_{\mu\nu}$$

$$h^A{}_\mu = 0 \text{ for } A \neq \mu, \qquad h^0{}_0 = 1 - \frac{m}{r}, \qquad h^i{}_k = \delta^i{}_k\left(1 + \frac{\chi m}{r}\right) \tag{1.9}$$

$$ds^2 = \left(1 - \frac{m}{r}\right)^2 dt^2 - \left(1 + \frac{\chi m}{r}\right)^2 (dx^2 + dy^2 + dz^2)$$

m is the geometrised mass of the central body, measured by the Kepler laws for his satellites. χ is a correction factor of the form $1 + 0(\max\phi)$. For stars like the sun we have $|1 - \chi| < 10^{-5}$. Light deviation is $\frac{1}{2}(1 + \chi)$ times the Einstein value and the perihelion motion is $\frac{1}{2}(1 + \frac{4}{3}\chi)$ times the Einstein value. Therefore the Einstein effects are nearly reproduced and the necessary conditions for a gravitation theory are fulfilled (Treder, 1967a; Liebscher, 1967).

The modification of the ordinary source density $T_\mu^{*\nu}$ by the field $h^A{}_\mu$ itself, i.e. the potential coupling of the sources, is the property discussed in this paper. In addition to the direct effect of absorption, changing the linear addition of potentials in the tetrad components (Treder, 1967a), an indirect effect also exists, arising from the fact that $h^0{}_0$ has to be squared to obtain the equivalent g_{00} of the Newton potential.

2. *The Absorption Effect in a Static Spherical Matter Distribution*

The static matter tensor of a fluid has the form

$$T_\mu{}^\nu = \begin{pmatrix} \rho, & 0, & 0, & 0 \\ 0, & -p, & 0, & 0 \\ 0, & 0, & -p, & 0 \\ 0, & 0, & 0, & -p \end{pmatrix} \tag{2.1}$$

Now we can put as tetrad and metric

$$\begin{gathered} \alpha_{\mu\nu} = \eta_{\mu\nu}, \\ h^0{}_0 = f, \qquad h^i{}_k = \delta^i{}_k g, \qquad h^i{}_0 = h^0{}_i = 0 \\ ds^2 = f^2\, dt^2 - g^2(dx^2 + dy^2 + dz^2) \end{gathered} \tag{2.2}$$

In the static spherical case, equations (1.7) reduce to

$$\Delta f = \kappa f \frac{\rho + 3p}{2} \tag{2.3}$$

$$\Delta g = -\kappa g \frac{\rho - p}{2}$$

The conditions in infinity $f = 1$ and $g = 1$ together with the matter condition $p < \rho$ restrict f and g to

$$f < 1 \qquad \text{and} \qquad g > 1 \tag{2.4}$$

We know from Einstein theory, that g_{00} corresponds to $1 - 2\phi$, ϕ being the Newton potential divided by c^2. Therefore we now have to compare f and $1 - \phi$ and later, more exactly, the resulting g_{00} with $1 - 2\phi$. If R is the radius of the spherical matter distribution, the outer solution of (2.3) is

$$f = 1 - \frac{m}{r} \tag{2.5}$$

$$g = 1 + \frac{\chi m}{r}$$

and we obtain for both masses

$$m = \frac{\gamma}{c^4} \int_0^R (\rho + 3p) . f . 4\pi r^2\, dr \tag{2.6}$$

$$\chi m = \frac{\gamma}{c^4} \int_0^R (\rho - p) . g . 4\pi r^2\, dr$$

Now we compare with the ordinary mass formula

$$M = \frac{\gamma}{c^4} \int_0^R \rho 4\pi r^2 \, dr \tag{2.7}$$

If we do not consider the pressure, the inequalities (2.4) cause

$$m < M < \chi m \quad (2.8$$

m being the active gravitational mass, we interpret (2.8) as an absorption of the source by the potential f. The source of g is antiabsorbed by g itself. The presence of the pressure causes changes in the opposite direction. The source density of f, diminished by f, is increased, and the source density of g, amplified by g, is decreased by the pressure.

To estimate these effects we consider an incompressible fluid: ρ constant. The dynamical equation for the matter tensor (2.1) reads, using the metric (2.2)

$$f \cdot \frac{dp}{dr} + (\rho + p)\frac{df}{dr} = 0 \quad (2.9)$$

and can be integrated, as can the following equation for f. But because the difference of m and M cannot be measured, but only the factor χ, we are mainly interested in χ and choose an iteration method to solve (2.9) and (2.3) together. We express f, g and p as series in $\frac{1}{2}\kappa\rho$. Substituting in (2.3) and (2.9) yields the coefficients successively.

$$p = \rho\left[\frac{R^2 - r^2}{6}\frac{\kappa\rho}{2} + \frac{3R^4 - 5R^2r^2 + 2r^4}{45}\left(\frac{\kappa\rho}{2}\right)^2 + 0\left(\left(\frac{\kappa\rho}{2}\right)^3\right)\right]$$

$$f = 1 - \frac{3R^2 - r^2}{6}\frac{\kappa\rho}{2} + \frac{5R^4 - r^4}{60}\left(\frac{\kappa\rho}{2}\right)^2 + 0\left(\left(\frac{\kappa\rho}{2}\right)^3\right) \tag{2.10}$$

$$g = 1 + \frac{3R^2 - r^2}{6}\frac{\kappa\rho}{2} + \frac{3R^4 - R^2r^2}{18}\left(\frac{\kappa\rho}{2}\right)^2 + 0\left(\left(\frac{\kappa\rho}{2}\right)^3\right)$$

We use these expressions in the mass formulas. Neglecting the terms of higher order we obtain the result

$$m = M\left(1 - \frac{3}{5}\frac{M}{R} + \frac{3}{7}\frac{M^2}{R^2} + \cdots\right) \tag{2.11}$$

$$\chi m = M\left(1 + \frac{M}{R} + \frac{27}{35}\frac{M^2}{R^2} + \cdots\right)$$

To isolate the effect of the pressure in these formulas one can use the series (2.10) in the formal case of $p \equiv 0$, equation (2.9) being neglected. Then we obtain

$$m = M\left(1 - \frac{6M}{5R} + 0\left(\frac{M^2}{R^2}\right)\right) \tag{2.12}$$

$$\chi m = M\left(1 + \frac{6M}{5R} + 0\left(\frac{M^2}{R^2}\right)\right)$$

So we are able to say that the effect of the pressure is of the same order as the effect of the potential coupling, but in general the latter prevails. Up to second order in the Newton potential the quotient χ of the both masses is greater than unity and the inequalities (2.8) are valid. The quotient χ, i.e. the real absorption, is not measurable in the case considered here, but the quotient χ is measurable, at least in principle, by light deflection and perihelion motion. m is given by the Kepler laws and M is not accessible.

The validity of the iteration method is doubtful only for dense stars, if M and R are of comparable magnitude. But if $R < 2M$, i.e. if the function f is less than $\frac{1}{2}$ on the surface and inside the body, then again the absorption effect prevails above the pressure, because the equation of state is conditioned by the inequality $\rho > 3p$. Therefore, considering (2.8), we have in this case $(\rho + 3p)f < \rho$ and $(\rho - p)g > \rho$.

For the sun we get $m = 1{\cdot}48.10^5$ cm, $R = 7.10^{10}$ cm and therefore $|\chi - 1| \sim 3.10^{-6}$.

3. *The Absorption Effect of the Mass of Earth*

The qualitative statement that the ordinary mass has to be multiplied by the potential $f = 1 - \Phi$ leads to the question as to how the mass of the earth behaves. Along the path around the sun, the distance between earth and sun changes, the factor $f = 1 - \Phi_{\text{sun}}$ changes, and this should affect the active gravitational mass of the earth. The variation of the solar potential is equal to the solar potential multiplied by the numerical eccentricity of the orbit. So we can expect a relative amplitude of 10^{-10} of the field of earth. Before carrying out a calculation we discuss some preliminary simplifications.

The earth being a freely falling body, we can state that the inner structure of the earth, being responsible for the pressure effect, is determined mainly by the field of the earth. Therefore its change can only be of the order of the absorption effect and can produce only higher orders in this effect. Therefore we neglect any effect of varying the inner structure, because the effect we search for is caused by the variation of the potential. In calculating a variation, we can also neglect the constant effects of inner structure and so we put as matter tensor of the earth

$$\underset{\mathrm{E}}{T^{\mu\nu}} = \rho u^\mu u^\nu, \qquad \rho \text{ constant} \tag{3.1}$$

ρ is here the mean density of the earth, deduced from the mean active gravitational mass:

$$\frac{\kappa\rho}{6} R_{\mathrm{E}}^{3} = M_{\mathrm{E}}$$

The dynamical equation, requiring the existence of some pressure, is not considered in the inner problem, because of the constancy of its effect. We consider only the outer dynamical equation, providing us with the Kepler motion.

The field equations of the tetrad are solved by expansion. The earth's field is considered as a disturbance of the solar field. Handled with care, we only need the first order. We denote by $\underset{0}{h}{}^{A}{}_{\mu}$ the unperturbed solar field given by (2.2) and (2.5), and by $\underset{1}{h}{}^{A}{}_{\mu}$ the perturbation produced by the earth. $\underset{\mathrm{S}}{T}{}^{\mu\nu}$ is the matter tensor of the sun and $\underset{\mathrm{E}}{T}{}^{\mu\nu}$ that of the earth. We write

$$\begin{aligned} &\Box \underset{0}{h}{}^{A}{}_{\mu} + \Box \underset{1}{h}{}^{A}{}_{\mu} \\ &\quad = -\kappa \left(\underset{0}{h}{}^{A}{}_{\nu} \underset{\mathrm{S}}{T}{}^{*\nu}_{\mu} + \underset{0}{h}{}^{A}{}_{\nu} \underset{\mathrm{E}}{T}{}^{*\nu}_{\mu} + \underset{1}{h}{}^{A}{}_{\nu} \underset{\mathrm{S}}{T}{}^{*\nu}_{\mu} + \underset{1}{h}{}^{A}{}_{\nu} \underset{\mathrm{E}}{T}{}^{*\nu}_{\mu} \right) \end{aligned} \tag{3.2}$$

The equation

$$\Box \underset{0}{h}{}^{A}{}_{\mu} = -\kappa \underset{0}{h}{}^{A}{}_{\nu} \underset{\mathrm{S}}{T}{}^{*\nu}_{\mu}$$

is solved by (2.2) and (2.5) outside the sun. $\chi - 1$ is of order 10^{-6} and can be neglected, because it produces obviously second-order effects. Designating by R the distance of the sun and by M its mass, we write

$$\underset{0}{h}{}^{0}{}_{0} = 1 - \frac{M}{R}, \qquad \underset{0}{h}{}^{i}{}_{k} = \delta^{i}{}_{k}\left(1 + \frac{\chi M}{R}\right), \qquad \underset{0}{h}{}^{0}{}_{i} = \underset{0}{h}{}^{i}{}_{0} = 0$$

The second part of equation (3.2),

$$\Box \underset{1}{h}{}^{A}{}_{\mu} = -\kappa \left(\underset{0}{h}{}^{A}{}_{\nu} \underset{\mathrm{E}}{T}{}^{*\nu}_{\mu} + \underset{1}{h}{}^{A}{}_{\nu} \underset{\mathrm{S}}{T}{}^{*\nu}_{\mu} + \underset{1}{h}{}^{A}{}_{\nu} \underset{\mathrm{E}}{T}{}^{*\nu}_{\mu} \right) \tag{3.3}$$

is solved only in the required approximation. At first we consider the several terms of the right-hand side. The last one describes the effect of the field of the earth on its own source. This effect is smaller than the effect of the solar potential and above it, it is nearly constant, the arguments being the same as in discussing the matter tensor of the earth. This term will be neglected. The second term on the right describes the effect of the potential of the earth on the source of the field of the sun. The result is of the same order of magnitude as the effect to be calculated, but in the neighbourhood of the earth the correction produced by this term has no component spherical around the earth. But any correction of the mass of the earth must appear as a spherical correction of the potential of the earth. This term will be cancelled. There remains

$$\Box \underset{1}{h}{}^{A}{}_{\mu} = -\kappa \underset{0}{h}{}^{A}{}_{\nu} \underset{E}{T}{}^{*\nu}_{\mu} \tag{3.4}$$

To avoid misunderstandings, it must be made clear that the mass of the earth is defined by the weight of the spherical component of the potential in the neighbourhood of the earth in the local rest system of the earth. It is because we measure the mass from the acceleration of a freely-falling body or from the period of a pendulum on the earth's surface. Measurements are made in the neighbourhood of the earth, therefore the distance of the sun is great in relation to the distance of the center of the earth. Corrections to the sources of the solar field, especially the exact form of the equation of state of the solar matter, and the inner field also, can produce only such corrections to the field, which do not contain spherical components in the neighbourhood of the earth. That the mass is defined by the spherical component of the potential is not necessary to discuss. But we must pay attention to the fact that we measure in the local rest frame. Therefore we calculate first the $\underset{1}{h}{}^{A}{}_{\mu}$ in the rest frame Σ of the solar system, then we transform into the local rest system Σ' of the earth and form g'_{00}, comparing it with $1-2\phi$. Because we need then only the spherical components of g'_{00}, it is sufficient to consider only the retarded potentials of the monopole components of the sources in the solutions of the equations for $\underset{1}{h}{}^{A}{}_{\mu}$.

In the neighbourhood of the earth we can consider the motion of the earth as quasistatic. The velocity is approximated to as constant on the momentary point of the orbit. We take the retardation into account only formally, because after transformation into the local rest system it disappears near the earth. The values of u^{μ} we find in the formulas of the Kepler motion in the solar field. The coordinate quadruple reads

$$x^{\mu} = \left(ct, \frac{p\cos\phi}{1+\varepsilon\cos\phi}, \frac{p\sin\phi}{1+\varepsilon\cos\phi}, 0 \right)$$

$$\frac{d\phi}{c\,dt} = \frac{L}{R^2}, \qquad R = \frac{p}{1+\varepsilon\cos\phi} \tag{3.5}$$

The rotation of the earth is not taken into account. The equation of motion yields

$$p = \frac{L}{M^2}, \qquad \frac{M}{p} = \frac{L^2}{p^2} \tag{3.6}$$

The four-velocity has the form

$$u^\mu = U\left(1, -\frac{L}{p}\sin\phi, \frac{L}{p}(\cos\phi+\varepsilon), 0\right)$$

$$U = \left[f^2 - g^2\frac{L^2}{p^2}(1+2\varepsilon\cos\phi+\varepsilon^2)\right]^{-1/2} \tag{3.7}$$

$$f = 1-\frac{M}{R}, \qquad g = 1+\frac{\chi M}{R}$$

Equation (3.4) can be written as

$$\Box \underset{1}{h^A}_\mu = -\kappa\rho\left(u_\mu \underset{0}{h^A}_\nu u^\nu - \tfrac{1}{2}\underset{0}{h^A}_\mu\right) \tag{3.8}$$

Again we neglect every term containing the correction $\underset{1}{h^A}_\mu$ together with the matter density ρ. Forming

$$T^{*\nu}_\mu = \rho u_\mu u^\nu - \tfrac{1}{2}\delta_\mu{}^\nu \rho g_{\alpha\beta} u^\alpha u^\beta$$

we put $g_{\mu\nu} = \eta_{AB}\underset{0}{h^A}_\mu \underset{0}{h^B}_\nu$. In this way we obtain (3.8) from (3.4).

The solution of (3.8) will be transformed into the local rest system Σ' and then composed to g'_{00}. We transform with a Lorentz transformation of velocity u^μ. The transformation matrix reads

$$\frac{\partial x^\mu}{\partial x'^\nu} = \begin{Bmatrix} \xi, & -\xi\frac{L}{p}\sin\phi, & \xi\frac{L}{p}(\cos\phi+\varepsilon), & 0 \\ -\xi\frac{L}{p}\sin\phi, & 1+0\left(\frac{L^2}{p^2}\right), & 0\left(\frac{L^2}{p^2}\right), & 0 \\ \xi\frac{L}{p}(\cos\phi+\varepsilon), & 0\left(\frac{L^2}{p^2}\right), & 1+0\left(\frac{L^2}{p^2}\right), & 0 \\ 0, & 0, & 0, & 1 \end{Bmatrix} \tag{3.9}$$

$$\xi = \left[1-\frac{L^2}{p^2}(1+2\varepsilon\cos\phi+\varepsilon^2)\right]^{-1/2}$$

Now we can write

$$g'_{00} = \eta_{AB}\left(\underset{0}{h^A}_\mu + \underset{1}{h^A}_\mu\right)\left(\underset{0}{h^A}_\nu + \underset{1}{h^A}_\nu\right)\frac{\partial x^\mu}{\partial x'^0}\frac{\partial x^\nu}{x\partial'^0} \tag{3.10}$$

To calculate the effect we look for we need the correction of the mass of earth to order $0(L^2/p^2) = 0(M/p)$. Because

$$\xi = 1+0\left(\frac{L^2}{p^2}\right), \qquad \frac{\partial x^i}{\partial x'^0} = 0\left(\frac{L}{p}\right)$$

we have to consider the corrections of $\kappa\rho$ only up to certain orders of L/p. Equation (3.8) reads explicitly

$$
\begin{aligned}
\Box \underset{1}{h}{}^{0}{}_{0} &= -\kappa\rho(U^2 f^3 - \tfrac{1}{2} f) \\
\Box \underset{1}{h}{}^{0}{}_{1} &= -\kappa\rho\left(\frac{L}{p}\sin\phi + 0\left(\frac{L^2}{p^2}\right)\right) \\
\Box \underset{1}{h}{}^{0}{}_{2} &= \kappa\rho\left(\frac{L}{p}(\cos\phi + \varepsilon) + 0\left(\frac{L^2}{p^2}\right)\right) \\
\Box \underset{1}{h}{}^{1}{}_{0} &= \kappa\rho\left(\frac{L}{p}\sin\phi + 0\left(\frac{L^2}{p^2}\right)\right) \\
\Box \underset{1}{h}{}^{2}{}_{0} &= -\kappa\rho\left(\frac{L}{p}(\cos\phi + \varepsilon) + 0\left(\frac{L^2}{p^2}\right)\right) \\
\Box \underset{1}{h}{}^{1}{}_{1} = \Box \underset{1}{h}{}^{2}{}_{2} &= \frac{\kappa\rho}{2}\left(1 + 0\left(\frac{L^2}{p^2}\right)\right) \\
\Box \underset{1}{h}{}^{1}{}_{2} = \Box \underset{1}{h}{}^{2}{}_{1} &= \kappa\rho 0\left(\frac{L^2}{p^2}\right)
\end{aligned}
\tag{3.11}
$$

The calculation of $\underset{1}{h}{}^{0}{}_{3}$, $\underset{1}{h}{}^{1}{}_{3}$, $\underset{1}{h}{}^{2}{}_{3}$ and $\underset{1}{h}{}^{3}{}_{\mu}$ is not necessary.

Now we write the formula (3.10) explicitly:

$$
\begin{aligned}
g'_{00} = {} & f^2\xi^2 - g^2\xi^2\frac{L^2}{p^2}(1 + 2\varepsilon\cos\phi + \varepsilon^2) \\
& + 2f\xi \underset{1}{h}{}^{0}{}_{\mu}\frac{\partial x^\mu}{\partial x'^0} \\
& + 2g\xi\frac{L}{p}\sin\phi\, \underset{1}{h}{}^{1}{}_{\mu}\frac{\partial x^\mu}{\partial x'^0} \\
& - 2g\xi\frac{L}{p}(\cos\phi + \varepsilon)\, \underset{1}{h}{}^{2}{}_{\mu}\frac{\partial x^\mu}{\partial x'^0}
\end{aligned}
\tag{3.12}
$$

We substitute

$$
\begin{aligned}
\underset{1}{h}{}^{0}{}_{\mu}\frac{\partial x^\mu}{\partial x'^0} &= \frac{2m}{r*}\xi\left[-U^2 f^3 + \frac{f}{2} + \frac{L^2}{p^2}(1 + 2\varepsilon\cos\phi + \varepsilon^2)\right] \\
\underset{1}{h}{}^{1}{}_{\mu}\frac{\partial x^\mu}{\partial x'^0} &= \frac{m}{r*}\xi\frac{L}{p}\sin\phi \\
\underset{1}{h}{}^{2}{}_{\mu}\frac{\partial x^\mu}{\partial x'^0} &= -\frac{m}{r*}\xi\frac{L}{p}(\cos\phi + \varepsilon)
\end{aligned}
\tag{3.13}
$$

Here we have denoted the retarded distance coordinate from the center of the earth by r^*. In the coordinates of the local rest system it is the usual distance coordinate. Now we obtain

$$g'_{00} = \xi^2\left[f^2 - g^2\frac{L^2}{p^2}(1 + 2\varepsilon\cos\phi + \varepsilon^2)\right] + \frac{2m}{r^*}\left[f^2\xi^2 - 2f^4U^2\xi^2 + 3\frac{L^2}{p^2}(1 + 2\varepsilon\cos\phi + \varepsilon^2)\right] \tag{3.14}$$

The correction factor of the mass of the earth is therefore

$$\theta = 2f^4U^2\xi^2 - f^2\xi^2 - 3\frac{L^2}{p^2}(1 + 2\varepsilon\cos\phi + \varepsilon^2)$$

Expanding in powers of L/p we find

$$\theta = 1 - \frac{2M}{p}(1 + \varepsilon\cos\phi) = 1 - \frac{2M}{R} \tag{3.15}$$

This is the desired result. The earth seems to be heavier at aphelion than at perihelion.

4. *Discussion*

From (3.15) we can see that the mass of the earth is as more absorbed as the distance from the sun decreases. The relative amplitude of the annual variation of the mass equals

$$\frac{2M}{p}.\varepsilon = 3{\cdot}3.10^{-10}$$

The acceleration in the field of the earth being 981 cm $\sec^{-2}$, we have to detect $6{\cdot}4.10^{-4}$ cm $\sec^{-2}$ with a measuring device which is stable for one year. Therefore the effect cannot yet be tested, but because it is only slightly beyond the present experimental accuracy one can hope for the near future.

It should be noted that we obtain the same result (3.14), if we consider the earth being at rest in the distance R from the sun. But the effect of the motion had to be calculated, because the square of the velocity has the same order of magnitude as the solar potential.

The potential coupling of the matter in (1.7) not only effects in absorption of mass, but it also induces higher poles. Under the technical assumption of a spherical body at rest in the solar field, one can see that there is a dipole induced, because the mass of the half turned towards the sun is more absorbed than the mass of the other half. This induced dipole is always directed towards the sun, independent of any rotation of the body. We could expect a daily period in the acceleration field of the earth at a fixed point on its surface, but the relative order of the amplitude of the variation is only 10^{-13}. This is far beyond the accuracy of measurements.

In the Einstein theory of gravitation we would not expect any effect of this kind. The strong equivalence principle being valid, the gravitational force between two freely falling bodies cannot depend on the position in a slowly-varying outer gravitational field, if the force in connection with the geodesic deviation in the outer field is not considered. Therefore in the Einstein theory the mass of the earth measured by terrestrial methods has to be constant. This statement does not mean that the total mass of a two-component system does not depend on the distance of the components, because here we denote as mass of the system the weight of the spherical part of the potential in the farther of the two bodies. The absorption of the 'far' mass of a system by its own field is in Einstein theory a fundamental property of gravitation. But the 'near' mass of one component should not vary. In this way a measurement of (3.15) can test Treder's equation (1.7) against the Einstein theory.

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